Wavefunction branches demand a definition!

Quantum Views **9**, 85 (2025) [arXiv:2506.15663]

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1 Dec 2025

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Many names for a constellation of problems

- “Preferred-basis problem”
- “Preferred-(sub)system problem”
- “Set-selection problem”
- “Quantum mereology”
- “Branch-definition problem”
- “Quantum-reality problem”

- Kent’s “quantum reality problem” is arguably the most general formulation:
  - What is the sample space for which quantum theory is supposed to determine probabilities?
  - e.g., configurations of beables, events, histories, paths, etc.
- In other words: what is the set of *things that can happen*?

- I this talk I will sketch a heuristic framework for conceptually organizing historical progress on the reality problem. I categorize by

    1. the (foundational/axiomatic) mathematical structure they *presume*

    2. the (emergent) mathematical structure they *derive*

- Naturally, this framework emphasizes my preferred criteria for an ultimate(-ish) solution:

    - presume only **spatial locality** as axiomatic structure on Hilbert space

    - derive a preferred time-dependent decomposition of the wavefunction of the universe $\psi(t)$ into **orthogonal branches**

- Progress since Birkhoff-von Neumann (1936, 89 years) or Everett (1957, 68 years) has been slow but steady!

    - I'll focus on work since proto-decoherence ideas of Zeh (1970), but I do think the framing is useful for thinking about earlier stuff, e.g., the generalization from *presumed measurement apparatus* to *presumed quasiclassical object*

- I'll briefly describe work by myself and by Weingarten towards a solution, complementing work by Taylor & McCulloch recently presented at this seminar

Trivial example: Preferred histories → preferred branches

- History "class" operator

$$C_\alpha = \hat{P}_{\alpha_N}(t_N) \cdots \hat{P}_{\alpha_1}(t_1)$$

with decoherence functional

$$D(\alpha, \beta) := \langle \psi | C_\alpha^\dagger C_\beta | \psi \rangle = p_\alpha \delta_{\alpha\beta}$$

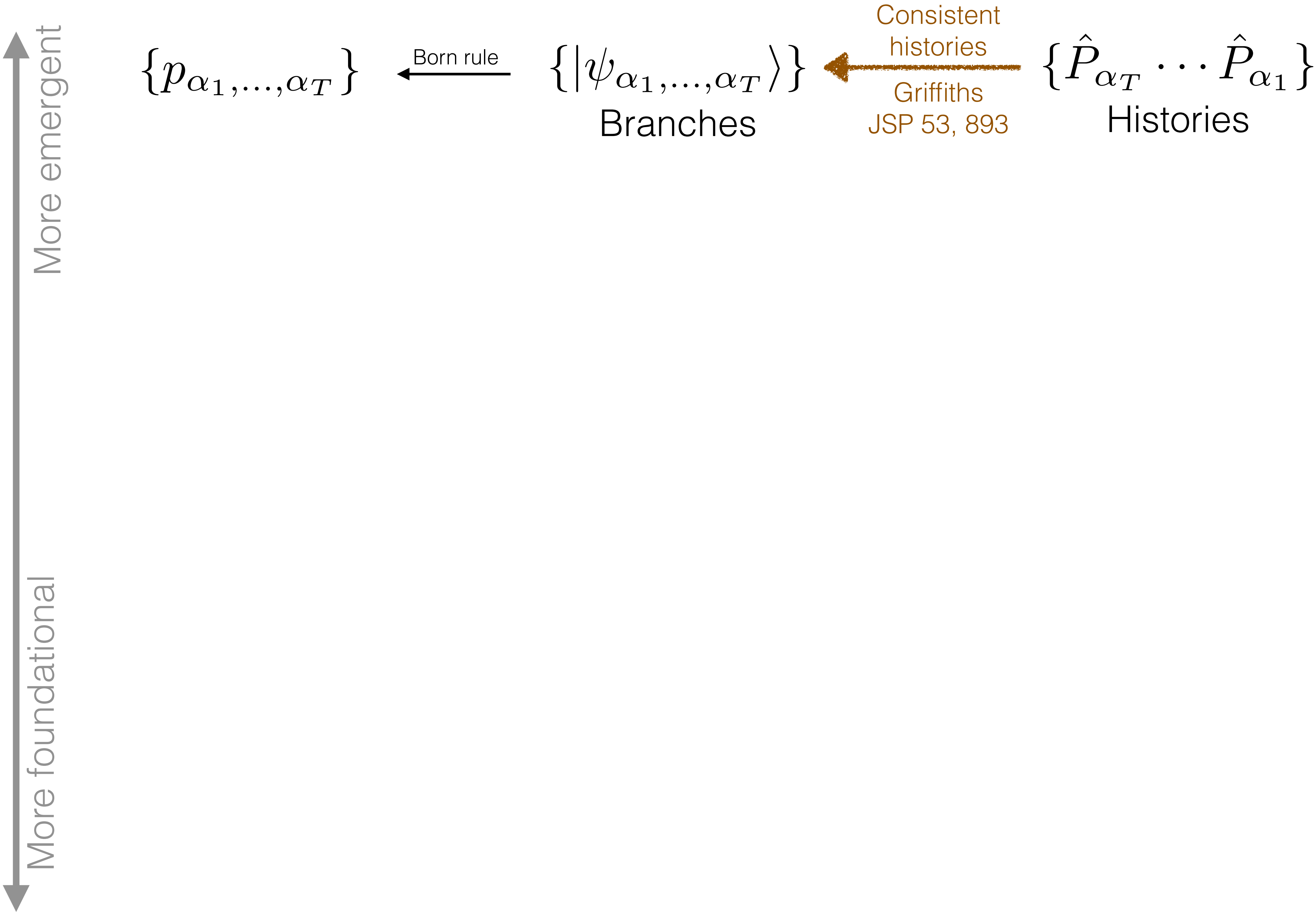
- These induce branches

$$|\psi_\alpha\rangle := C_\alpha |\psi\rangle$$

which are orthogonal

$$\langle \psi_\alpha | \psi_\beta \rangle = p_\alpha \delta_{\alpha\beta}$$

- But opposite direction is one-to-many
- Consistent histories "recovers" Everett



$\{p_{\alpha_1,\ldots,\alpha_T}\}$
Born rule
$\{|\psi_{\alpha_1,\ldots,\alpha_T}\rangle\}$
Branches
Consistent histories
Griffiths JSP 53, 893
$\{\hat{P}_{\alpha_T}\cdots\hat{P}_{\alpha_1}\}$
Histories
More emergent
More foundational

Decoherence: subsystem → pointer states

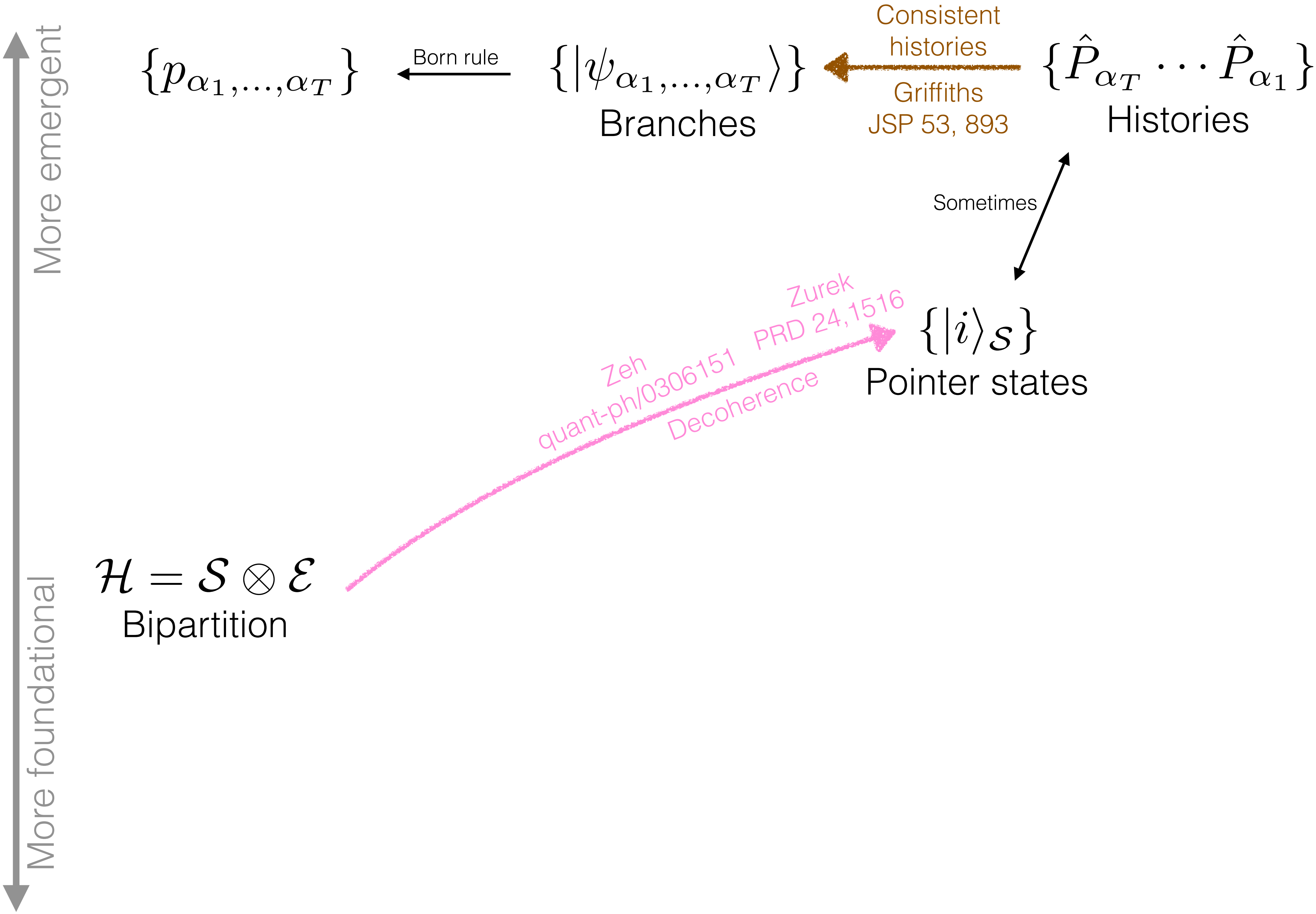
- Crudely, decoherence theory (Zeh, Zurek)

  1. *presumes* some preferred (sub)system $\mathcal{S}$; and

  2. *derives* a preferred set of pointer states $|i\rangle_{\mathcal{S}}$

- Basic decoherence theory does not tell us what the system $\mathcal{S}$ we should focus on

- But the classical limit and desiderata of **predictability** (Zurek, Carroll & Singh) suggest we focus our attention on systems that are **Markovian** and **slow**, properties *determined by the Hamiltonian* $\hat{H}$



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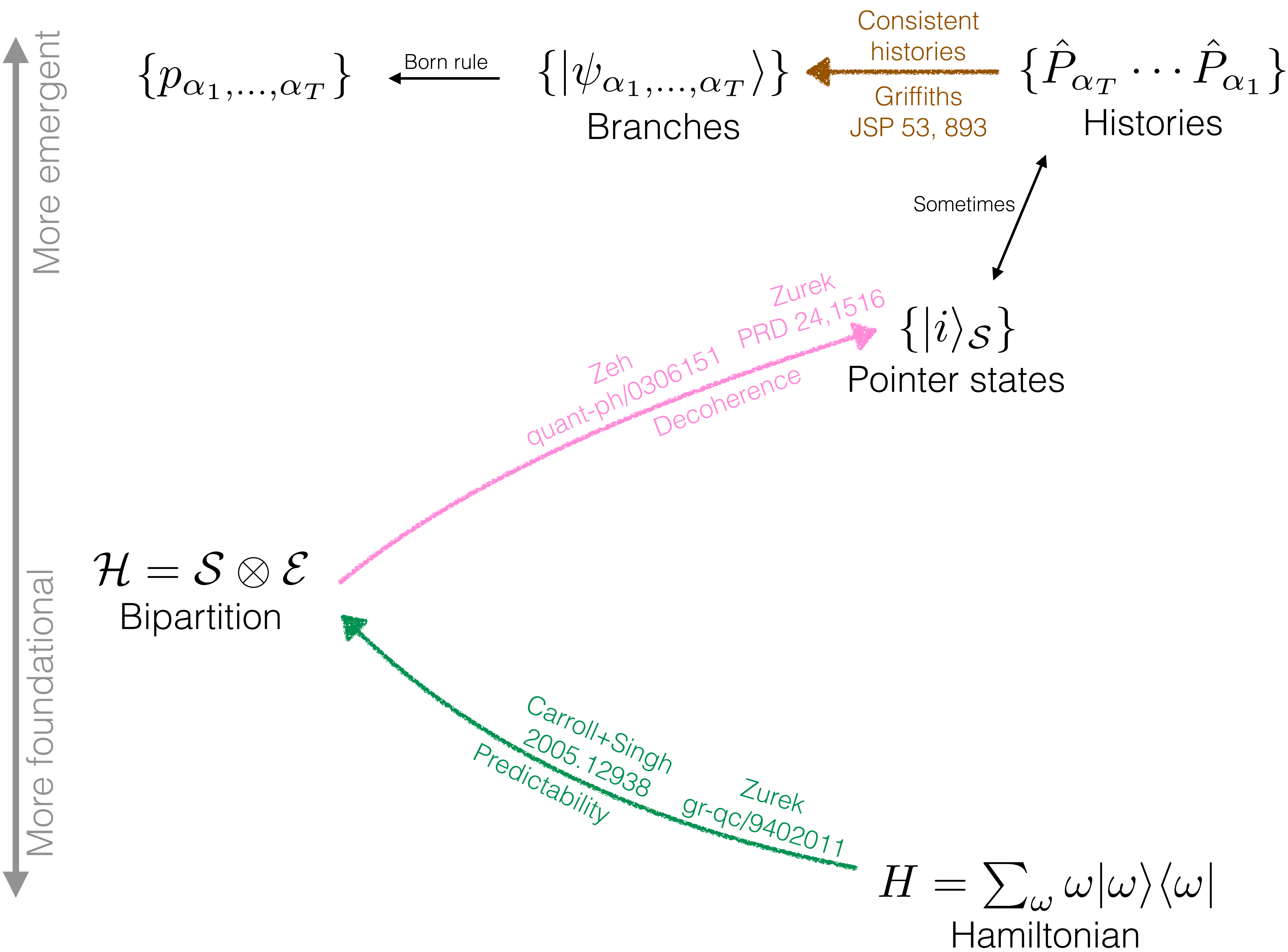
$\{|i\rangle_{\mathcal{S}}\}$
Pointer states
Zurek PRD 24,1516
Zeh quant-ph/0306151
Decoherence
$\mathcal{H}=\mathcal{S}\otimes\mathcal{E}$
Bipartition



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Carroll+Singh 2005.12938
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Zurek gr-qc/9402011
$H=\sum_\omega \omega|\omega\rangle\langle\omega|$
Hamiltonian

Preferred hydro observable → preferred histories

- Consider a fixed observable

$$\hat{A} = \sum_i a_i \Pi_i$$

that is **hydrodynamic**: it has approximately Markovian equations of motion because

  - it is the mesoscopic average of a density obeying a local continuity equation, so
  - it evolves very slowly compared to microscopic degrees of freedoms which
  - thermalize rapidly compare to evolution of the hydro variable

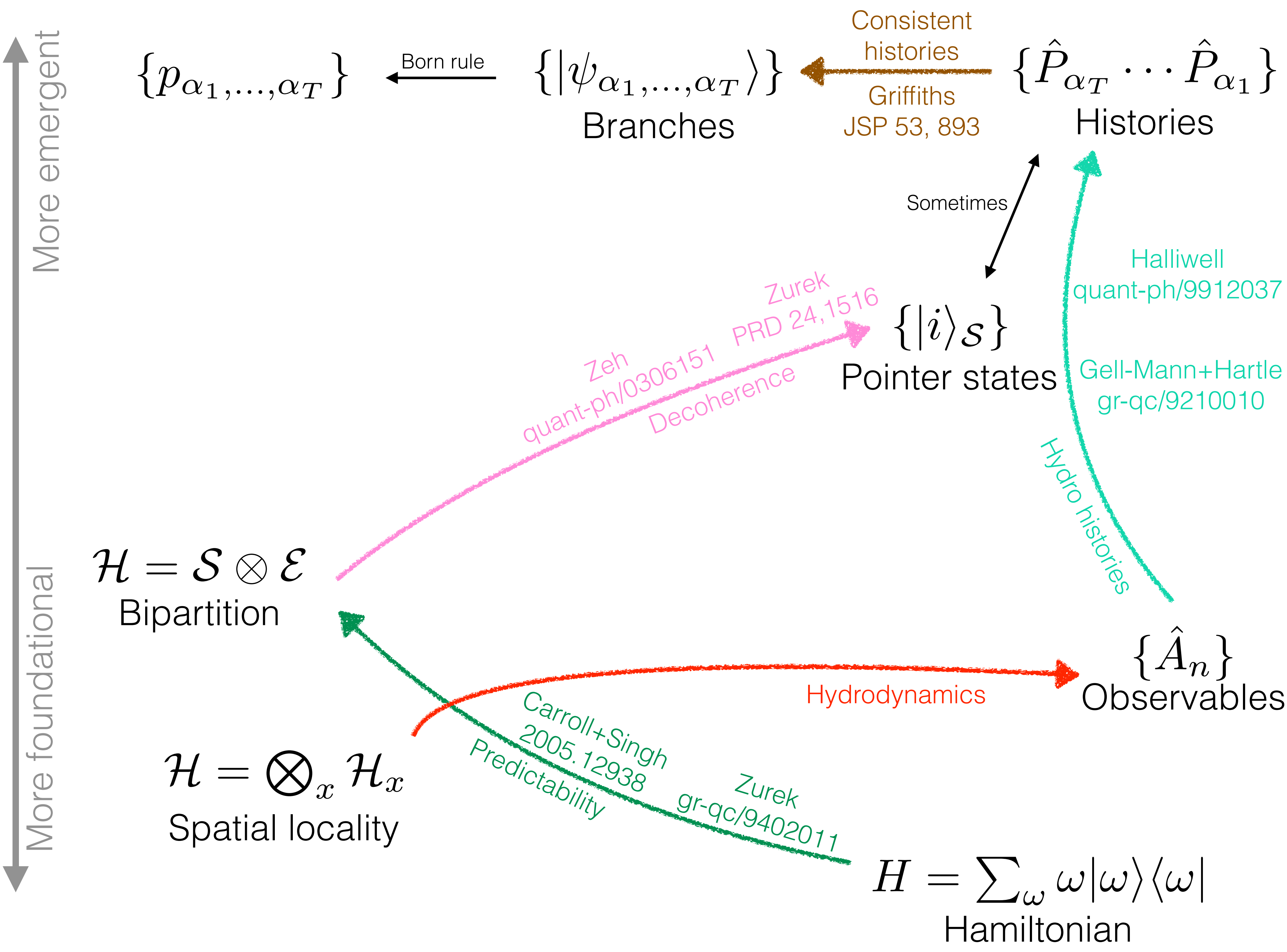
- Then generally (by Halliwell and Gell-Mann & Hartle), for appropriate coarse-graining timescale τ and coarse-graining hydro scale δa, the coarse-grained histories of this observable are consistent:

$$C_\alpha = \hat{P}_{\alpha_N}(t_N) \cdots \hat{P}_{\alpha_1}(t_1) \qquad t_{n+1} - t_n = \tau$$

$$\hat{P}_{\alpha_n} = \sum_{i:|a_i - a(\alpha_n)| < \delta a} \Pi_j$$

$$D(\alpha, \beta) = \langle \psi | C_\alpha^\dagger C_\beta | \psi \rangle = p_\alpha \delta_{\alpha\beta}$$

- Hydro variables are crucially *local* average: picked out by **spatial locality**



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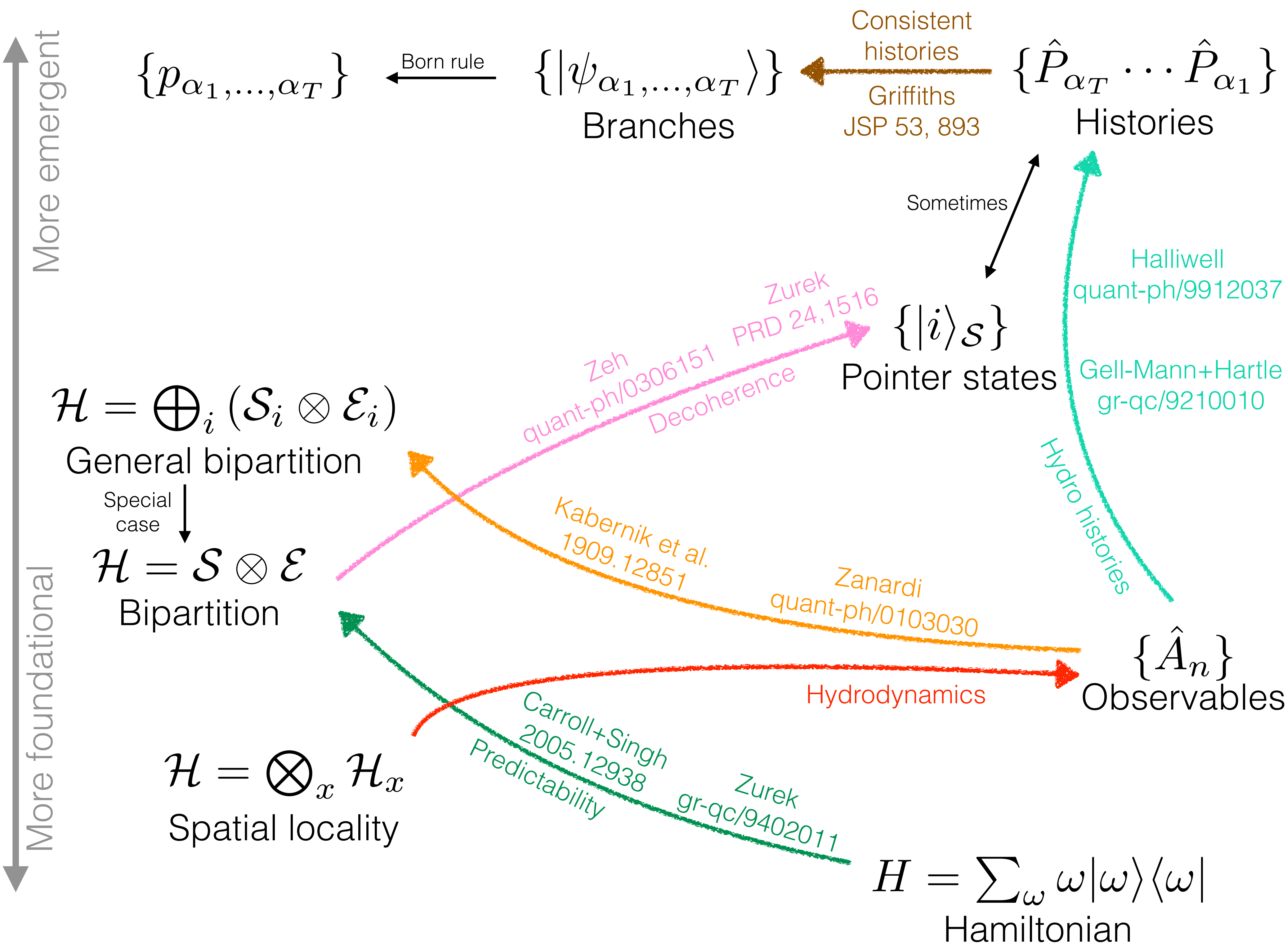

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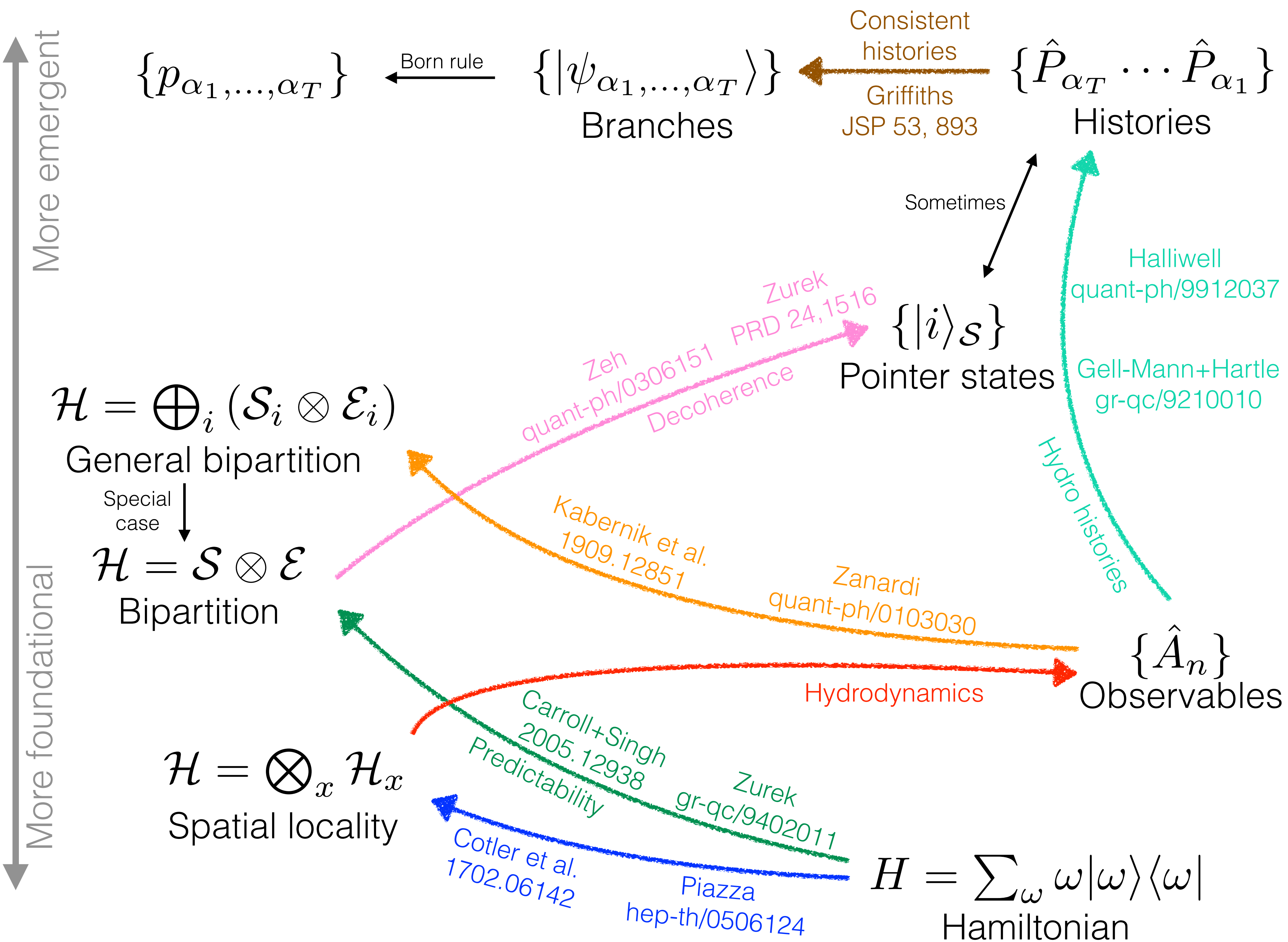
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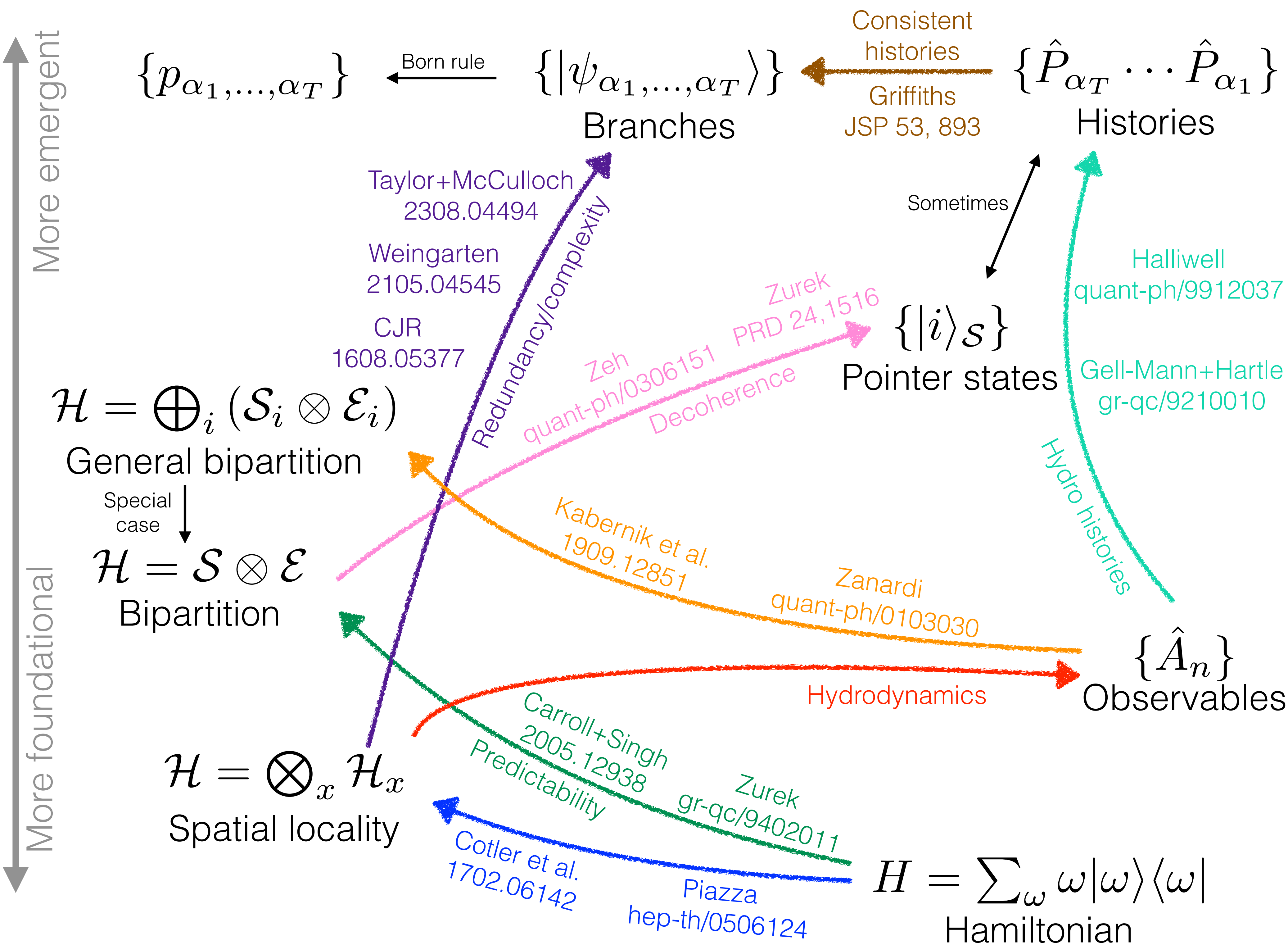

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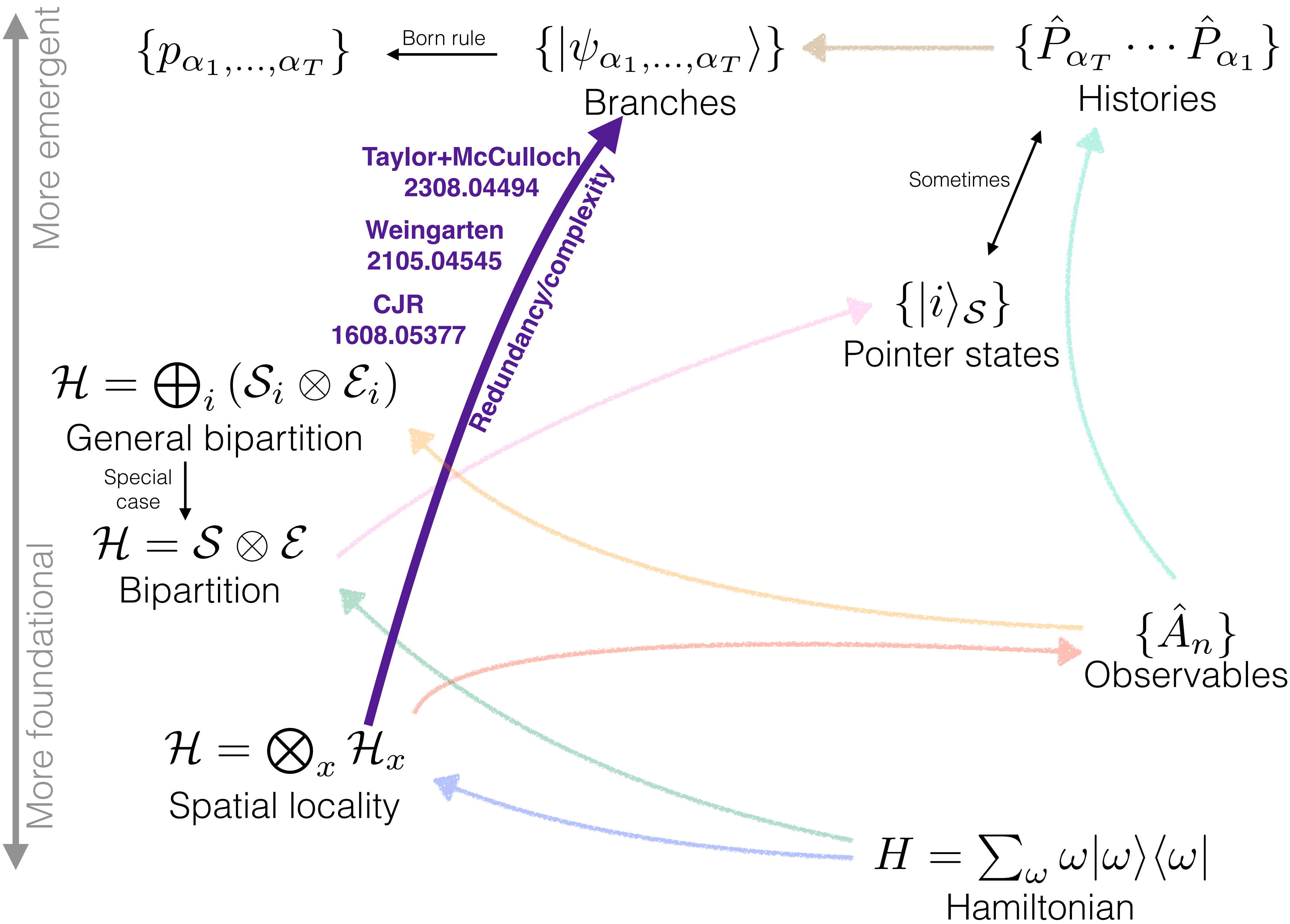

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Defining (many-body) branches

- As mentioned, there are different names for slightly different formulations of the quantum reality problem based on *what is assumed* and *what is derived*

- "Defining many-body branches" is just shorthand for *assuming spatial locality* and *deriving an orthogonal decomposition*

$$\mathcal{H} = \bigotimes_x \mathcal{H}_x \longrightarrow \{|\psi_{\alpha_1,\ldots,\alpha_T}\rangle\}$$

Spatial locality → Branches

Defining (many-body) branches

$$\mathcal{H} = \bigotimes_x \mathcal{H}_x \longrightarrow \{|\psi_{\alpha_1,\ldots,\alpha_T}\rangle\}$$

Spatial locality → Branches

- This is essentially the most fundamental version of the quantum reality problem, with these caveats:
    - You might think the spectrum is more fundamental than spatial locality (a philosophical question), but if you buy Cotler et al. then the former recovers the latter.
    - You might think something else (neither spatial locality or the spectrum) is more fundamental. I haven't seen viable proposals, but I'm all ears.
    - It's nonrelativistic. A fundamental solution would be relativistic, relying instead on *spacetime locality* (an assumption of QFT). My framing with the nonrelativistic case is just the way to get started. Weingarten shows a potential way to generalize to relativistic case.
    - It might be *strategically* helpful to think about operators (history projectors, or delocalized algebras), rather than orthogonal decompositions, but the former recovers the latter.

Three approaches

- I think there have been basically three attempts to solve this so far

    1. CJR as simultaneous eigenstates of *records* [1608.05377]

        - See also follow-up by Ollivier [2202.06832]

    2. Wiengarten: minimize difference of *average branch complexity* and *decomposition entropy* [2105.04545]

        - See also follow-up by Wiengarten [2308.05569]

    3. Taylor & McCulloch by maximizing difference of *interference complexity* and *distinguishability complexity* [2308.04494]

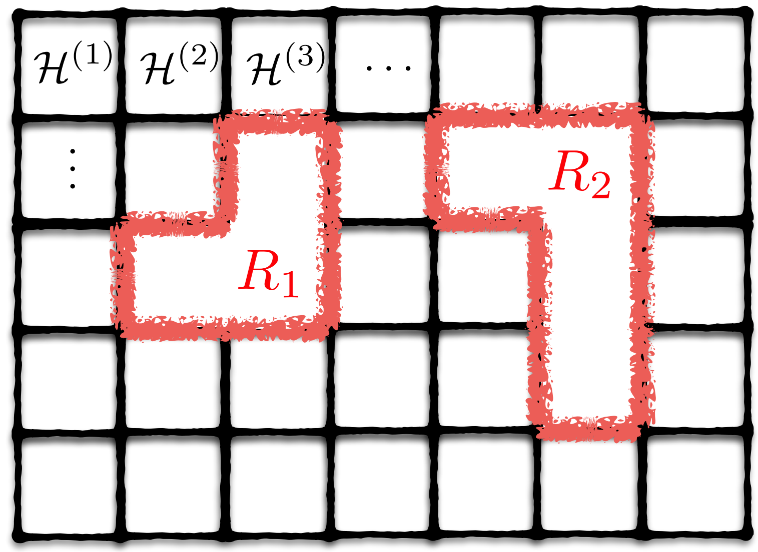
- You’ve heard about Taylor & McCulloch so I will talk mostly about CJR and Wiengarten

Defining *recorded PVMs*

- Assume a state ψ on a lattice (any microscopic tensor-product structure):

$$\psi \in \mathcal{H} = \bigotimes_n \mathcal{H}^{(n)}$$

- A region is subsets of the lattice:



- A projection-valued measure on a region (local PVM) is a complete set of orthogonal projectors localized to the region:

$$(P_i^R)^\dagger = P_i^R = P_i^R \otimes I^{\bar{R}}$$

$$\sum_i P_i^R = I \qquad P_i^R P_j^R = \delta_{ij} P_i^R$$

- Two local PVMs on disjoint regions record each other (with respect to ψ) when their projectors are fully (classically) correlated:

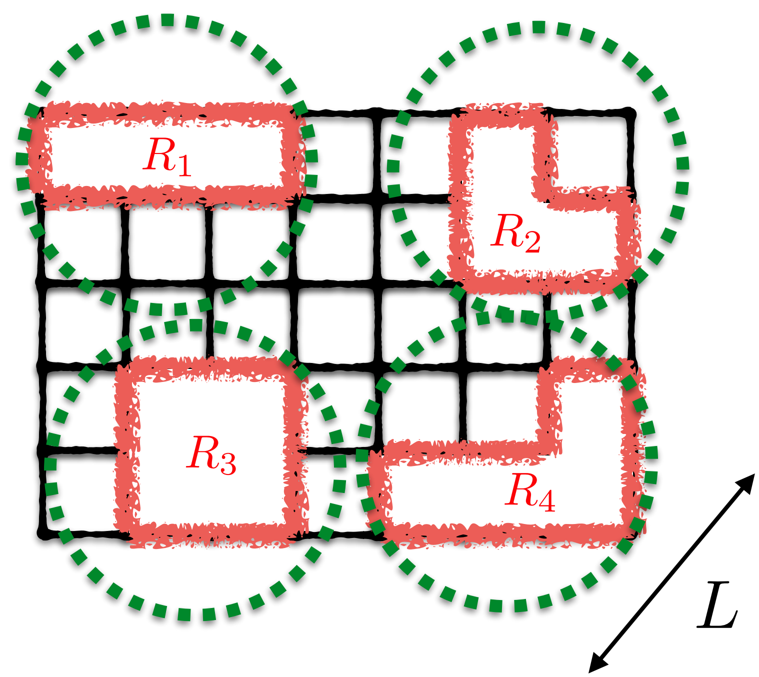
$$P_i^{R_1} |\psi\rangle = P_i^{R_2} |\psi\rangle$$

Defining *redundant record*

- Let a redundant record be a set of 3 or more local PVMs that all record each other

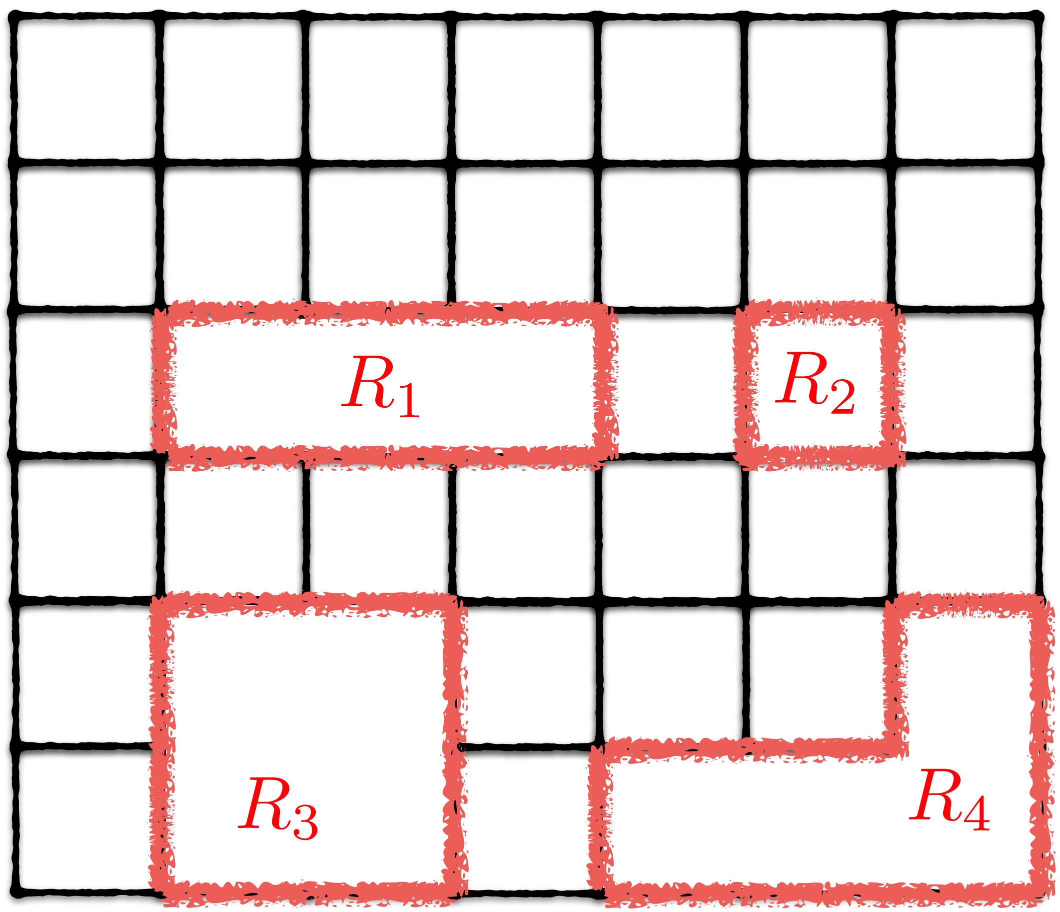
$$P_j^{R_1}|\Psi\rangle = P_j^{R_2}|\Psi\rangle = P_j^{R_3}|\Psi\rangle = \cdots$$

- Let a redundant record at scale L be a redundant record where the regions fit in disjoint spheres of diameter L



Compatible recorded observables

- Now consider *multiple* redundant records, e.g., for different classical macroscopic outcomes



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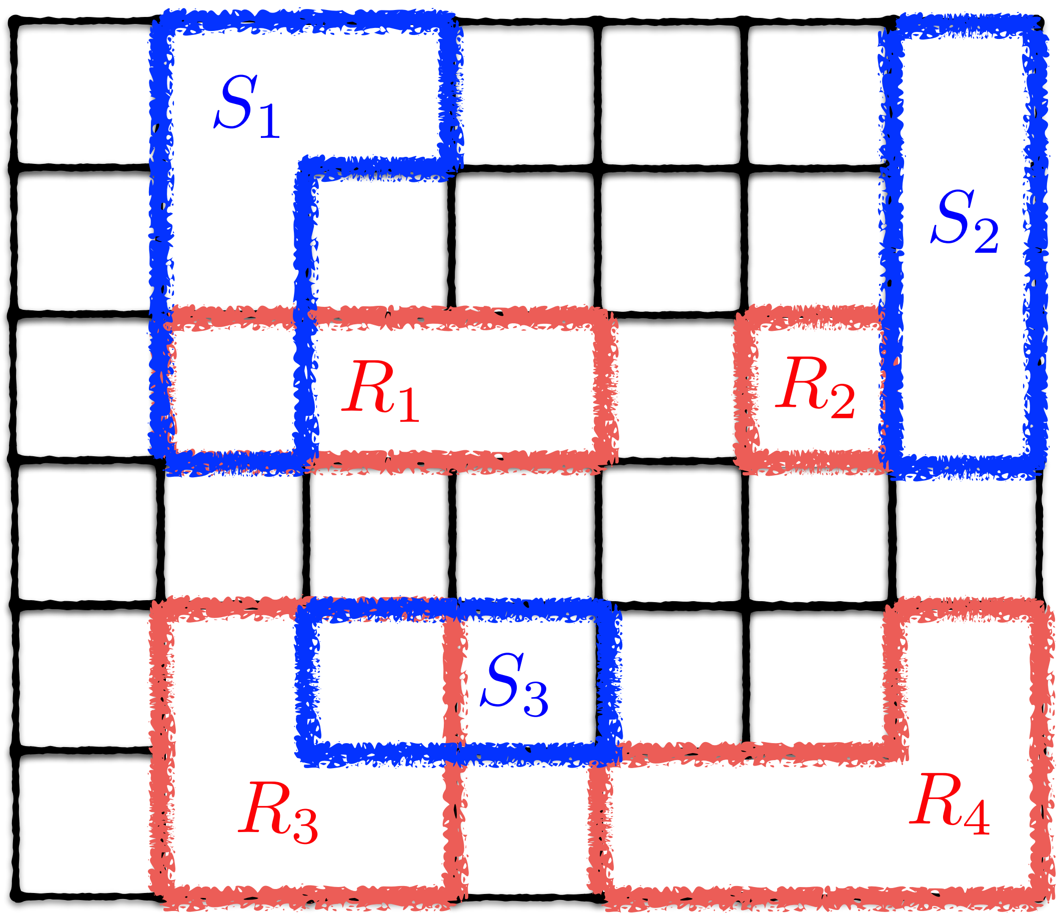

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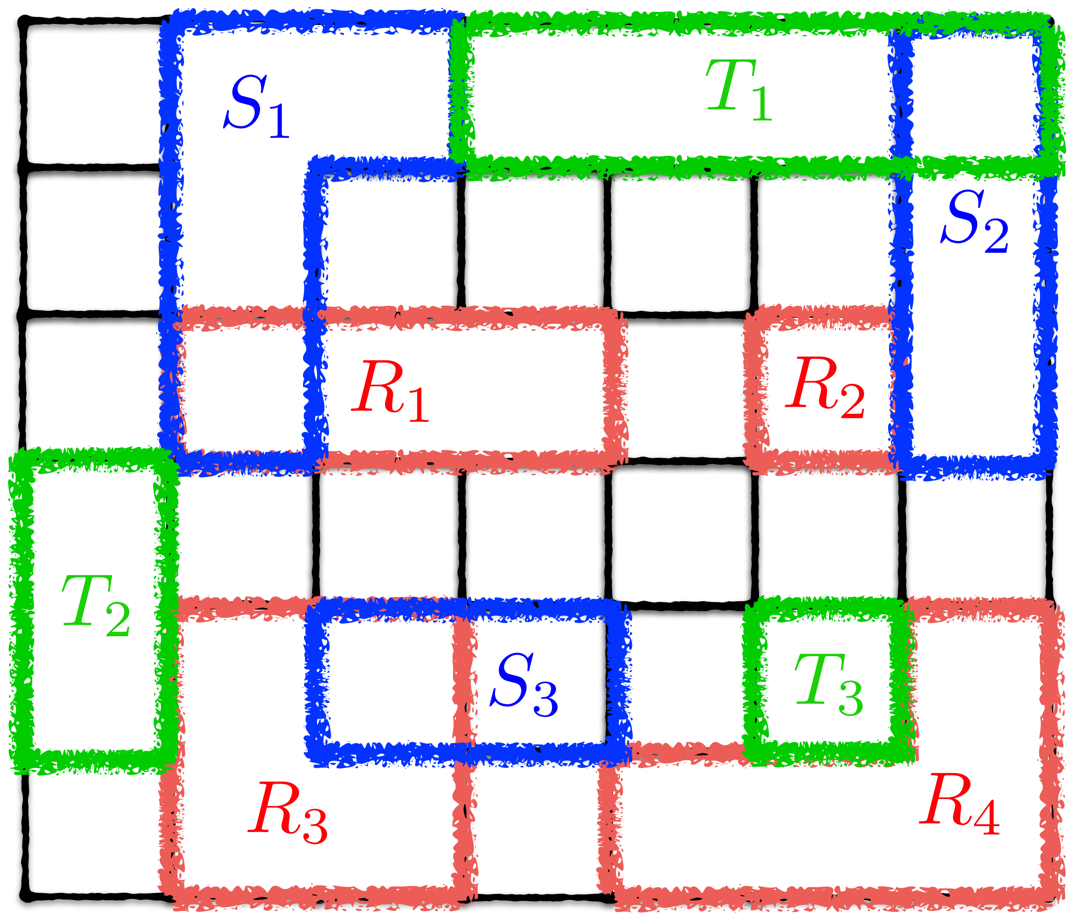
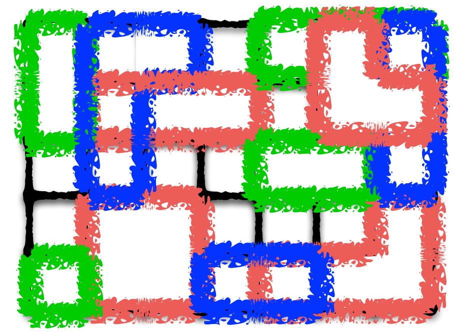


Uniqueness of branch structure at fixed scale



- **Theorem**: All projectors in all PVMs of all redundant records at the same scale are necessarily commuting *on the state*:

$$[Q, Q']\psi = 0,$$

Generated algebra (Products and linear combos)

Outcomes

Regions

$$Q, Q' \in \langle \{P_i^{R_a}\}_{i,a}, \{P_j^{S_b}\}_{j,b}, \{P_k^{T_c}\}_{k,c}, \cdots \rangle$$

- Define redundancy branches at scale L to be the simultaneous eigenstates:

$$\psi_{(i,j,k,\ldots)} := P_i^{R_a} P_j^{S_b} P_k^{T_c} \cdots \psi$$

$$\psi = \sum_{I=(i,j,k,\ldots)} \psi_I = \sum_i \sum_j \sum_k \cdots \psi_{(i,j,k,\ldots)}$$

Comments on redundancy branches

- For $L\approx$10 cm, these branches are eigenstates of “everything known to at least three human brains”
  - Different L can in principle give different decompositions, although it requires ψ to be a really weird error-correction-code type state
- This is not viable fundamental candidate mainly because
  - L-dependence is ugly
  - It doesn’t capture irreversibility; (huge) matter interferometers are incorrectly “branched”
- But shows that it’s possible to construct a preferred decomposition based only many-body entanglement structure *with no reference to a preferred system*

Complexity for branches

- Both Weingarten and Taylor & McCulloch use quantum state complexity to construct branches
    - (Weingarten uses Nielsen complexity and Taylor & McCulloch use traditional circuit complexity, but I will elide the difference.)
- They rely crucially on some form of the the generic asymptotic linear growth of complexity ("2nd law of quantum complexity"), forms of which have very recently been proven rigorously.
    - This is a quantum correlate of the generic linear growth of entropy in classical non-integrable systems
- Definition: Fix a set of "elementary" unitary operations. The **relative complexity** $C(\phi,\chi)$ between state vectors ϕ and χ is the *minimum* number of elementary operations needed to compose a unitary operator U that maps ϕ and χ up to some accuracy $\Delta \in [0,1]$:

$$|\langle\phi|U|\chi\rangle| \geq \Delta$$

    - On a lattice, the elementary operations are usually N-qubit operators (e.g., N=2), thereby encoding the notion of spatial locality
    - There is yet no principled choice single obvious choice of elementary set in the continuum limit relevant to QFT, but Brown et al. have some results suggesting the renormalization-group ideas dramatically narrow down the plausible candidates

Weingarten branches

- Weingarten propose that the proper branch decomposition of a state ψ is the orthogonal decomposition $\psi = \sum_\alpha \psi_\alpha$ that minimizes a weighted sum of the expected squared complexity of the branches and the Shannon entropy of their squared norms:

$$
\begin{aligned}
Q(\{\psi_\alpha\}) &:= \sum_\alpha |\psi_\alpha|^2 \left[C(\psi_\alpha, \Omega)^2 - b|\psi_\alpha|^2 \right] \\
&= \bar{C}^2(\{\psi_\alpha\}) + b\, S(\{\psi_\alpha\})
\end{aligned}
$$

- Here,
  - Ω is the QFT vacuum state
  - $\bar{C}^2(\{\psi_\alpha\}) := \sum_\alpha |\psi_\alpha|^2 C(\psi_\alpha, \Omega)^2$ is the mean squared branch complexity
  - $S(\{\psi_\alpha\}) := -\sum_\alpha |\psi_\alpha|^2 \ln |\psi_\alpha|^2$ is the Shannon entropy of the branch decomposition
  - b is weakly constrained new constant of nature

Weingarten branches

$$Q(\{\psi_\alpha\}) := \sum_\alpha |\psi_\alpha|^2 \left[C(\psi_\alpha, \Omega)^2 - b|\psi_\alpha|^2\right]$$
$$= \bar{C}^2(\{\psi_\alpha\}) + b\,S(\{\psi_\alpha\})$$

- Q is better motivated than it first appears:
  - We want a way of decomposing the highly entangled global state into branches that are less entangled, but we don't want to simply decompose it into product states with zero entanglement: there is a trade off between complicatedness of the decomposition and the complicatedness of the components
  - Complexity is used because, by the 2nd law, *that's the thing that reliably increases under unitary time evolution*
  - Shannon entropy (rather than, e.g., Rényi entropy) and the additive form are used to guarantee independent branching on uncorrelated regions

Weingarten branches

- The decomposition is manifestly invariant under translations, rotation, and internal symmetries

- Weingarten gives evidence that…

  - The definition can be made Lorentz covariant by using Kent's asymptotic late-time construction

  - The decomposition is stable in the continuum limit! This allows for a branch decomposition in quantum field theory by taking the limit of lattice QFT

  - Branches are exceedingly unlikely to recombine at a later time; first case of using 2nd law of complexity to capture irreversibility in branching

Weingarten branches

- Issue #1: Depends on a preferred length scale b. Weingarten proposes this as a new constant of the universe, but I am very skeptical
- Issue #2: Still lots of work to be done to tell whether this actually recovers existing knowledge of branches, e.g., all of decoherence theory
- Issue #3: Inefficient to compute. Would need heuristics to use for simulation
- Nevertheless, I think this is a very impressive result that should be studied carefully. Read it!

D. Weingarten, "Macroscopic Reality from Quantum Complexity" *Foundations of Physics* 52, 45, (2022) [2105.04545]

Objections to branches generally

- Objection: such-and-such interpretation of QM says I don't have to worry about this
    - Response: OK, forget interpretation if you must. I am pointing to a *beautiful* mathematical phenomenon with *global* applicability. Let's understand it!
- Objection: This doesn't have applications
    - Response: There's reason to think this could allow classical simulation of *real world* quantum systems
    - Jordan Taylor presented our numerical evidence; it will be in his thesis and our forthcoming paper

Best objection

- The best objection I hear: this is *too hard*
- Yes. It looks hard.
- But hardly anyone has tried! This is not quantum gravity or P≠NP!
- The prize is potentially *immense*: removing a fundamental ambiguity in the axioms of QM, speeding up numerical simulations, and understanding beautiful ubiquitous structure
- If you think it's *soooooo hard* that no progress can be made, then prove a no-go theorem! Or just elucidate the problems with existing attempts.
- Consider: do the objections about how hard it must be to make progress apply to pre-decoherence quantum mechanics?

The End

CJR, "Wavefunction branches demand a definition!" *Quantum Views* 9, 85 (2025) [2506.15663]

CJR, "Classical branch structure from spatial redundancy in a many-body wavefunction", *Physical Review Letters* 118, 120402 (2017) [1608.05377]

D. Weingarten, "Macroscopic Reality from Quantum Complexity" *Foundations of Physics* 52, 45, (2022) [2105.04545]

J. Taylor & I. McCulloch, "Wavefunction branching: when you can't tell pure states from mixed states", *Quantum* 9, 1670 (2025) [2308.04494]