Pointer states

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Some history

From Lorentz to Schrödinger 1926

> *"But a wave packet can never stay together and remain confined to a small volume in the long run. [...] Because of this unavoidable blurring a wave packet does not seem to me to be very suitable for representing things to which we want to ascribe a rather permanent individual existence."[1]*

Schrödinger's answer: The continuous transition from micro- to macro-mechanics 1926 [2].

From Schrödinger to Einstein 1950

"The conception of a world that really exists is based on there being a far-reaching common experience of many individuals, in fact of all individuals who come into the same or a similar situation with respect to the object concerned."[1]

"1. definition (Objectivity). A state of the system S exists objectively if many observers can find out the state of S independently, and without perturbing it."

Roads to objectivity: Quantum Darwinism, Spectrum Broadcast Structures, and Strong quantum Darwinism – a review [3]

Pointer states

Laboratory Quantum Mechanics

There is a quantum system and a classical world.

The two are separated by the so called Heisenberg cut.

The quantum system evolves according to the Schrödinger equation.

But then when we measure it, the wave function collapses to one of the eigenstates of the operator that the measurement device measures.

The measurement result will be the eigenvalue of that eigenstate.

The probability of this outcome is the absolute squared of the overlap of the quantum state of the system and that eigenstate.

Problems: two types of time evolution, the measurement which is a complex process, appears in the laws, the measurement device also consists of atoms, the observer is not described, we can not do cosmology this way.

We have only two axioms:

- The states of the system are elements of a Hilbert space.
- The time evolution is unitary.

The other "axioms" of quantum mechanics follow:

- When we measure the system, the wave function collapses to one of the eigenstates of the operator that the measurement device measures.
- The measurement result will be the eigenvalue of that eigenstate.
- The probability of this outcome is the absolute squared of the overlap of the quantum state of the system and that eigenstate.

Observables

Neumann measurement: $|S_1\rangle$, $|S_2\rangle$ are two states of the measured system. $|M_R\rangle$ is the ready state of the device.

$$|S_1\rangle\,|M_R\rangle \xrightarrow{t} |S_1\rangle\,|M_1\rangle$$

$$|S_2\rangle\,|M_R\rangle \xrightarrow{t} |S_2\rangle\,|M_2\rangle$$

$$(\alpha\,|S_1\rangle + \beta\,|S_2\rangle)\,|M_R\rangle \xrightarrow{t} \alpha\,|S_1\rangle\,|M_1\rangle + \beta\,|S_2\rangle\,|M_2\rangle$$

Let's calculate the norm of the two sides![4] Under unitary time evolution the norm does not change.

$$\langle S_1|S_2\rangle = \langle M_1|M_2\rangle\,\langle S_1|S_2\rangle$$

Either the two states $|S_1\rangle$ and $|S_2\rangle$ are orthogonal, or if they are not, we can divide by $\langle S_1|S_2\rangle$, and then $|M_1\rangle = |M_2\rangle$ follows, so this is not a successful "measurement", the state of the device does not reflect the state of the system.

But it does not follow, that any Hermitian operator corresponds to an observable quantity.

A general vector in the product Hilbert state

If we have two subsystems $H = H_1 \otimes H_2$, $\{|\phi_i^1\rangle\}_{i=1}^n$ in H_1, $\{|\phi_i^2\rangle\}_{i=1}^n$ in H_2 orthonormal bases, then a general $|\psi\rangle$ in the product Hilbert space can be written as:

$$|\psi\rangle = \sum_{i,j} \alpha_{ij} |\phi_i^1\rangle |\phi_j^2\rangle ,$$

but there exists an orthonormal basis, the Schmidt basis, in which:

$$|\psi\rangle = \sum_{i} \sqrt{\mathrm{p}_i} |\phi_i^1\rangle |\phi_i^2\rangle$$

In the general case, if $H = H_1 \otimes H_2 \otimes H_3 \otimes \ldots$

$$|\psi\rangle = \sum_{i,j,k,\ldots} \alpha_{ijk\ldots} |\phi_i^1\rangle |\phi_j^2\rangle |\phi_k^3\rangle \ldots ,$$

and there are no tricks to decrease the number of summations.

Branching state:

$$|\psi\rangle = \sum_{i} \alpha_i |\phi_i^1\rangle |\phi_i^2\rangle |\phi_i^3\rangle \ldots$$

Environmental decoherence [5]

If we have a small dust particle (s), and it will interact with photons (ε), first they are not entangled: $\hat{\rho}(0) = \hat{\rho}_s(0) \otimes \hat{\rho}_\varepsilon(0)$. In the beginning: $\psi = \sum_x \psi(x) |x\rangle$ After the first scattering:

$$|x\rangle |\chi_1\rangle \to \hat{S} |x\rangle |\chi_1\rangle = |x\rangle |\chi_1(x)\rangle ,$$

the back-reaction is neglected, $|\chi_1(x)\rangle$ the state of the first photon after scattering, which contains some information about the place of the dust particle. After scattering, the first photon goes by, and the next photon arrives. After N scatterings:

$$|\psi_{s\varepsilon}\rangle = \sum_x \psi(x) |x\rangle |\chi_1(x)\rangle |\chi_2(x)\rangle \dots |\chi_N(x)\rangle$$

$$\rho_s = \mathrm{Tr}_\varepsilon |\psi_{s\varepsilon}\rangle \langle\psi_{s\varepsilon}| = \sum_{x,x'} \rho(x,x',0) |x\rangle \langle x'| \langle\chi_1(x')|\chi_1(x)\rangle ... \langle\chi_N(x')|\chi_N(x)\rangle$$

All $\langle\chi_i(x')|\chi_i(x)\rangle < 1$, if $x \neq x'$, the off-diagonal part $\to 0$

Three cases of decoherence

$$H = H_s + H_{\text{int}} + H_\varepsilon$$

Certain states of the system can keep their purity in spite of their interaction with the environment. These are called pointer states.

- $H_s \ll H_{\text{int}}$: measurement limit
- $H_s \gg H_{\text{int}}$: quantum limit
- $H_s \approx H_{\text{int}}$: general case

- H_{int} eigenstates
- H_s eigenstates
- coherent states

Searching for pointer states [8]

Pointer states: states of the system that keeps their purity despite the interaction with the environment.

L. Diósi, C. Kiefer: Robustness and diffusion of pointer states [6].

N. Gisin, M. Rigo: Relevant and irrelevant nonlinear Schrodinger equations [7]

We would like to distinguish some states of the Hilbert space, we will need a non-linear equation for this. We describe the system as an open quantum system:

$$\dot{\rho}_s = \widehat{\mathcal{L}}(\rho_s), \quad \text{where } \widehat{\mathcal{L}} \text{ is a superoperator}$$

We are searching for those pure states who's projector (P) can maintain their purity during the non-unitary time evolution:

$$\dot{P} \approx \widehat{\mathcal{L}}(P), \quad \|\dot{P} - \widehat{\mathcal{L}}(P)\|_{HS} = \min$$

Searching for pointer states cont.

Change of a projector: $X = X^+$

$$P_{t+\delta t} = P_t + \delta t[P_t, [P_t, X]] = P_t + \delta t(PX - 2PXP + XP)$$

$$\begin{aligned} P^2_{t+\delta t} &= P^2_t + \delta t(PPX - 2PPXP + PXP + PXP - 2PXPP + XPP) + \delta t^2 \ldots \\ &= P_t + \delta t(PX - 2PXP + XP) = P_{t+\delta t} \end{aligned}$$

$$P^+_{t+\delta t} = P_{t+\delta t}$$

For a given $\widehat{\mathcal{L}}(P) = Z$, how should we choose X to keep $\|\widehat{\mathcal{L}}(P) - \dot{P}\|_{HS} = \mathrm{minimal}$?

$$\begin{aligned} \mathrm{Tr}[(Z - [P_t, [P_t, X]])^2] = Tr[Z^2 - 2(Z^2 P_t - (ZP_t)^2)] + \\ +2\mathrm{Tr}[(Z - X)^2 P_t - ((Z - X)P_t)^2] \end{aligned}$$

First term independent of X, second term minimal if $Z = X$

$$\dot{P}_t = [P_t, [P_t, \widehat{\mathcal{L}}(P)]]$$

Searching for pointer states cont.

If $\widehat{\mathcal{L}}$ is in Lindblad form:

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + \sum_i \gamma_i \left(L_i \rho L_i^+ - \frac{1}{2}\{L_i^+ L_i, \rho\} \right)$$

And we substitute this into our non-linear equation we get:

$$|\dot{\psi}\rangle = -\frac{i}{\hbar}(H - \langle H \rangle) |\psi\rangle + \gamma \left[\langle L^+ \rangle (L - \langle L \rangle) - \frac{1}{2}(L^+ L - \langle L^+ L \rangle) \right] |\psi\rangle$$

If $L = L^+$ we can simplify this to:

$$|\dot{\psi}\rangle = -\frac{i}{\hbar}(H - \langle H \rangle) |\psi\rangle + \gamma \left[(L - \langle L \rangle)^2 - \langle (L - \langle L \rangle)^2 \rangle \right] |\psi\rangle$$

Example 1: Measurement

$$L = L^{+} = A, \qquad H_s \approx 0$$

$$|\dot{\psi}\rangle = \frac{\gamma}{2}\left((A - \langle A\rangle)^2 - \langle (A - \langle A\rangle)^2\rangle\right)|\psi\rangle$$

This is purely decohering, the only possibility to stay pure is $|\dot{\psi}\rangle = 0$. If $|\psi\rangle$ is an eigenvector of A, the left side is 0.

For a measurement, the pointer states are the eigenvectors of the measured operator.

Example 2: Damped harmonic oscillator

$$H_s = \omega \mathrm{a}^\dagger \mathrm{a} \qquad\qquad L = a$$

The system can loose energy during the interaction with the environment. Coherent state:

$$|\alpha\rangle = \mathrm{e}^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

This is an eigenvector of the lowering operator: $\mathrm{a}\,|\alpha\rangle = \alpha\,|\alpha\rangle$.
Substituting this to the nonlinear equation makes it linear:

$$|\dot{\alpha}\rangle = \left((-i\omega - \frac{\gamma}{2})\alpha \mathrm{a}^\dagger + \frac{\gamma}{2}|\alpha|^2\right) |\alpha\rangle$$

$$|\alpha\rangle_t = |\alpha_0 \mathrm{e}^{i\omega t - \gamma t/2}\rangle$$

These states are not orthogonal!

$$|\langle\alpha|\beta\rangle|^2 = \mathrm{e}^{-|\alpha-\beta|^2}$$

Predictability

Pointer states are not so well defined in the literature, there are several definitions.

Zurek's predictability sieve: entropy production of pointer states should be minimal.

Let's maximize predictability (minimize Neumann entropy).
Or linear entropy, that is purity.

$$\partial_t S_{lin}[\rho] = \partial_t \mathrm{Tr}[\rho^2] = 2\mathrm{Tr}[\rho\dot{\rho}] = 2\mathrm{Tr}[\rho\widehat{\mathcal{L}}(\rho)] = \min$$

Pointer states are labeled by parameters. Predictability $\rightarrow$ pointer states' time evolution in this parameter space avoid each other $\rightarrow$ emergence of some kind of phase space?

Objectivity and Redundancy [9]

The environment weakly measures the system.

We observe fragments of the environment when we want to learn about the system: the fragments contain records (information) about the system.

The state of the system is objective when many *independent* observers agree about it.

An isolated quantum system's state can not be objective.

Nothing can make every property of a quantum system objective, since some observables are incompatible.

Pointer observables are those that can become objective.

Records about the pointer observables has to be stored redundantly in the environment fragments.

Summary

Summary

- The system and the environment evolves into branching states under unitary time evolution.
- In this process redundant records are created in the environment fragments.
- Meanwhile the system decoheres into a mixture of robust pointer states.

This is the dynamics of objectivity in an emergent multiverse.

Questions?

Questions

- In the Zurek or Diósi-Kiefer method we have to first tell apart the system and the environment. Quantum mereology.
- In the decoherent histories approach I think we ask for too much: we do not necessarily need decoherence in the environment.
- UHU. Parameter counting for local Hamiltonians. $\rightarrow$ Order from Chaos paper.
- General quantum states decohere to mixture of pointer states. But we did not prove that with the right weights. Can one prove it generally?
- If yes, what follows from that derivation for over-complete pointer states?
- If that calculation does not use trace, are we closer to the derivation of the Born-rule? Indirect measurement.

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