Alternative views of the Aharonov-Bohm effect

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The original Aharonov-Bohm paper

Experimental setup - electric

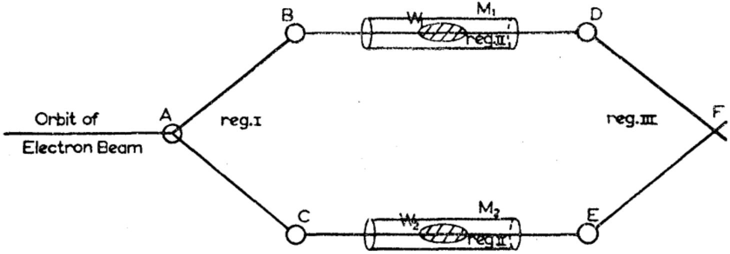

FIG. 1. Schematic experiment to demonstrate interference with time-dependent scalar potential. A, B, C, D, E: suitable devices to separate and divert beams. W_1, W_2: wave packets. M_1, M_2: cylindrical metal tubes. F: interference region.

Experimental setup - magnetic

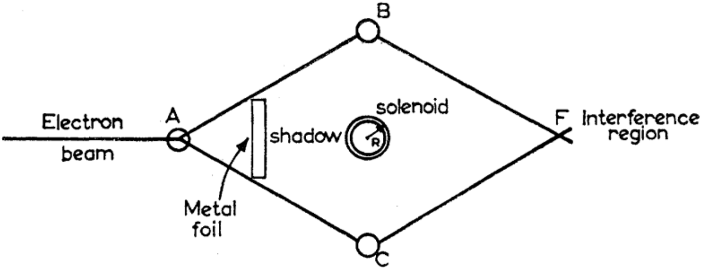

FIG. 2. Schematic experiment to demonstrate interference with time-independent vector potential.

Y. Aharonov and D. Bohm Phys. Rev. 115, 485, 1959

Significance of Electromagnetic Potentials in the Quantum Theory

- There is a physical effect of the potentials even though no force is ever actually exerted on the electron.
- $(S_1 - S_2)/\hbar = (e/\hbar c)\oint \mathsf{A} \cdot d\mathsf{x} = (e/\hbar c)\phi_0$
- Contact of the electron with the magnetic field can be avoided.
- In a field-free multiply connected region of space the physical properties of the system still depend on the potentials.
- These effects of the potentials depend only on gauge-invariant quantities.
- All fields interact locally, so one cannot interpret such effects as due to the fields themselves.

AB cont.

- In classical mechanics potentials cannot have such significance because the equation of motion involves only the field quantities themselves.
- We thought that the potentials have been regarded as purely mathematical auxiliaries, only the fields were thought to have direct physical meaning.
- In quantum mechanics we use the Schrödinger equation instead of the equation of motion. That cannot be expressed by the fields alone, we need the potentials.
- So in quantum mechanics the fundamental physical entities are the potentials.
- These examples show that if we want to maintain local interactions only, the potentials must be considered as physically effective.

Alternative views

Role of potentials in the Aharonov-Bohm effect

Argues that there is an alternative to the commonly accepted mechanism which leads to the effect, and that we might change our understanding of the nature of physical interactions back to that of the time before the AB effect was discovered.

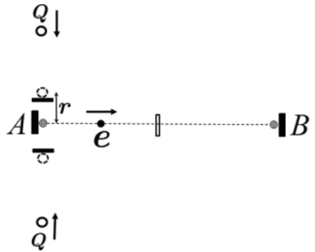

FIG. 2. A realization of the electric AB effect. Identical charges brought symmetrically to the electron wave packet in the left arm of the interferometer create a potential for the electron without creating an electric field at its location.

Vaidman cont.

- The electron does not experience an electric field at any place.
- The quantum state of the composite system is a superposition of two product states. The energy in the left branch is equal to the energy in the right branch.
- The electron's wave packets are not shifted.
- The charges Q, do experience different forces in different branches.
- Simplified calculation for the magnetic case.
- The solenoid is made of circling charged particles.
- The solenoid has quantum nature as well!
- The AB phase shift is caused by the electron's electric field on the solenoid.

Vaidman cont. 2

- The standard formulation of quantum mechanics, and the Schrödinger equation in particular, are based on potentials. I hope that a general formalism of quantum mechanics based on local fields will be developed.
- I have not presented a general proof that in order to have an observable effect, that particles must pass through regions of nonzero fields. Rather, what I have shown is that the setups of the electric and magnetic AB effects do not contradict this assertion.

Y. Aharonov, E. Cohen, D. Rohrlich: Phys. Rev. A **92**, 026101, 2015

- Six other thought experiments, where they claim Vaidman's idea does not work.

L. Vaidman: Phys Rev. A **92**, 026102, 2015

Quantized vector potential and alternative views of the magnetic Aharonov-Bohm phase shift

- The exact problem involves three quantized quantities: the electron, the solenoid charges, and the vector potential.

$$i\frac{d}{dt}|\psi,t\rangle = \left[\hat{H}_{\text{el}} + \hat{H}_{\text{sol}} + \hat{H}_A - \int d\mathbf{x}[\hat{\mathbf{J}}_{\text{el}}(\mathbf{x}) + \hat{\mathbf{J}}_{\text{sol}}(\mathbf{x})] \cdot \hat{\mathbf{A}}(\mathbf{x})\right]|\psi,t\rangle,$$

- Three truncated problems: two of these entities are considered classical, one quantum in the interaction term.

Pearle-Rizzi cont.

- Standard treatment: $\hat{\mathsf{J}}_{sol}(x) \to \mathsf{J}_{sol}(x)$, $\hat{\mathsf{A}} \to \mathsf{A}_{sol}$, $H_{int} = \int d\mathsf{x}\, \hat{\mathsf{J}}_{el}(\mathsf{x}) \cdot \mathsf{A}_{sol}(\mathsf{x}) \to$ correct AB shift. Only the elctron is quantized, there are no fields, the vector potential's effect on the electron illustrates the marvelous distinction between classical and quantum physics.

- Vaidman's idea: $\hat{\mathsf{J}}_{el}(x) \to \mathsf{J}_{el}(x)$, $\hat{\mathsf{A}} \to \mathsf{A}_{el}$, $H_{int} = \int d\mathsf{x}\, \hat{\mathsf{J}}_{sol}(\mathsf{x}) \cdot \mathsf{A}_{el}(\mathsf{x})$. Only the quantized solenoid charges undergo interaction $\to$ correct AB shift.

- Third possibility: only the vector potential is quantized: $H_{int} = \int d\mathsf{x}\, [\, \mathsf{J}_{el}(\mathsf{x}, t) + \mathsf{J}_{sol}(\mathsf{x})] \cdot \hat{\mathsf{A}}(\mathsf{x}) \to$ correct AB shift.

Pearle-Rizzi cont. 2

There are three other problems: when only one entity is kept classical. One of these is when only the vector potential is kept classical. Naive calculation $\rightarrow$ double the AB shift.
Better approximation, vector potential expressed with solenoid and electron operators, Schrödinger equation from variational principle, additional phase $\rightarrow$ correct AB shift.

Exact problem, all three entities are quantized.

- Variational principle, three Schrödinger equations
- In each one entity interacts with classical (expectation) values of the other two entities plus an additional phase term.
- There is a freedom of adding phases to two equations and compensate it in the third one.
  $\rightarrow$ correct AB shift.

Summary

Summary

- Heisenberg cut: we can put it several ways, but with caution.
- Aharonov-Bohm conclusion that the potential has physical effect when there are no fields was wrong (I think).
- Vaidman was right, AB shift can be explained by the electron acting on the particles of the solenoid.
- Non-locality vanishes when we quantize the electromagnetic potential as well.
- This method is quite general when we have two interacting things, should we use it in other cases as well?
- Is it possible to have a theory without potentials? As Vaidman hoped.
- We started from a classical theory and quantized it. In that classical theory there is the problem of the ”probe” charge. We see now that even one electron can cause phase shifts on the source of the E field. What if this is the tricky part?