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Internal Absorption Properties of Accelerating Detectors

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The existence of electromagnetic radiation from a uniformly accelerated charge has appeared in some recent work to present a problem suggesting the inadequency of the equivalence principle. For a proper treatment of the problem, it is desirable to show how the absorption properties of detectors are effected by being physically attached to noninertial frames of reference. An invariant criterion of internal absorption is formulated, and is identified with the observable behavior of an elementary detector. It is shown that the properties of a detector fixed in a uniformly accelerated frame are different from the properties of an inertial detector of similar construction, and that this difference is consistent with the usual form of the equivalence principle.

I. INTRODUCTION

In 1920, Pauli (*1*) argued that a uniformly accelerated electric charge does not radiate energy, and that the classical expression for radiation rate $\frac{2}{3}e^2a^2/c^3$ (where a is proper acceleration) applies only to a bound or periodically moving charge e. Pauli's argument is based on the circumstance that at the instant a uniformly accelerated charge comes to rest in a given inertial frame (the turning point of its hyperbolic motion), the magnetic field is everywhere zero, and hence the Poynting vector is everywhere zero. Since a Lorentz transformation can reduce any point of a world line to rest, Pauli concludes that the radiation from any point on the world line of a uniformly accelerated charge vanishes.

The limitations of this argument were first noted by Drukey (*2*) and have been further clarified by Bondi and Gold (*3*) and Fulton and Rohrlich (*4*). BG and FR are in apparent agreement (for essentially different reasons) that energy is radiated from a uniformly accelerated charge, but their separate agruments will not be restated here.

Bondi and Gold draw the further conclusion that the gravitational principle of equivalence cannot be generally applied to radiation phenomena. Their argument proceeds in two steps. First, it is recognized that a static charge in a static gravitational field cannot radiate, for otherwise energy would have to be transferred through the surrounding space (including the flat space at large distances from real gravitating masses) by time independent fields. Second, we are asked to restrict attention to the local uniform gravitational field of the charge, and consider a freely falling coordinate frame relative to which the charge is uniformly accelerating. According to the principle of equivalence, this falling frame is an inertial system in which Maxwell's equations are valid; and so the accelerating charge must be radiating energy to its surroundings. We therefore have a case in which radiation exists in one (inertial) frame but not in another (gravitational) frame. This is claimed to be paradoxical because the observation of radiation should be invariant. To solve the paradox, these authors suppose that the local gravitational field (the first order limit of the equivalence principle) must be contained in the local electromagnetic field where radiation of this origin is not detectable. In the wave zone of this radiation, it is supposed that higher order gravitational effects just compensate the electromagnetic field in such a way as to prevent detection.

In the present paper we assume that a uniformly accelerated charge in an inertial frame does radiate relative to that frame, and that the equivalence principle can be extended into the wave zone as far as the usual gravitational approximation permits. These assumptions do not imply conflicting detector observations, for we are able to show that detectors physically attached to different reference frames have different absorption properties. In particular, we show that a detector at rest in a static uniform gravitational field will not absorb internal energy from an electric charge held static in that frame, whereas a free falling detector may (generally) absorb internal electromagnetic energy from such a field.

The argument here rests on the claim that the invariant absorption properties of a detector are dependent on the constraints which physically attach it to a given reference frame. This question is essentially untouched in the literature, and so in Section II we explain the method and criterion to be used. In Sections III and IV, we apply the method to the case of the uniformly accelerated reference frame given by the Møller transformation equations (*5*), and arrive at the results stated above in terms of the equivalent gravitational field.

In Section V, the method of this paper is applied to the physically different case of a charge which is free falling in a uniform gravitational field, or static in the associated inertial frame. Our conclusion in this case is that a free falling detector will not absorb internal energy from such a charge, but that a detector held stationary in the gravitational field will (generally) absorb internal energy. Again, this can be given an invariant meaning only if one includes an account of the constraints which act in the latter case. Rohrlich (*6*) has arrived at similar conclusions, but without making use of an invariant criterion to identify the observable absorption characteristics of detectors.

II. INTERNAL ABSORPTION

We are concerned with deciding when an electromagnetic field in a given region of space-time is capable of delivering energy to a given detector in that region. This ought to have an invariant meaning identifiable with real transitions in the internal state of the detector. The absorption criterion we use for this purpose is classical and refers to the properties of an "elementary detector" which can simulate the absorption behavior of more complex detectors in the same region.

The elementary detector consists of a pair of + and − charges which are constrained to move over some specified closed path in the region of interest. The charges are assumed to begin together at some space-time point A, and to move over independent paths from there to a second point of coincidence B, thereby closing a "detector loop." If one observes such a detector loop from a covering inertial system, then the total electromagnetic four-momentum absorbed by the charges $+q$ and $-q$ in a single cycle is given by:

$$\begin{aligned} \Delta p_\mu &= q \int_A^B F_{\mu\nu}\, d\lambda^\nu(\text{path } {}^+q) - q \int_A^B F_{\mu\nu}\, d\lambda^\nu(\text{path } {}^-q) \\ \Delta p_\mu &= q \oint F_{\mu\nu}\, d\lambda^\nu \end{aligned} \tag{2.1}$$

where $F_{\mu\nu}$ is the electromagnetic field tensor. The positive direction around the loop is given by the motion of the + charge.

The proposed invariant criterion now states that if the impulse Δp_μ is timelike, then the specified detector loop has absorbed internal energy,[1] and if it is spacelike, then it has not absorbed internal energy. The null case is special, but it is invariantly meaningful to say that the fourth component of a null four-vector is or is not zero. In the former case we say that internal energy has not been absorbed, and in the latter case that it has been absorbed. Our assumption is that if the electromagnetic field is at all capable of delivering internal energy to any detector in a given spacetime region, then it should be possible to construct an elementary detector loop in that region that yields a timelike four-vector Δp_μ or a null four-vector Δp_μ having nonzero components.[2] The work done against radiation reaction, and the external forces which constrain the charges to move over any specified loop is not included because it is not relevant to the energy transfer capability of the electromagnetic field by itself.

In an inertial frame of reference, this criterion adequately describes what we mean by internal absorption; for if the impulse Δp_μ received in any cycle is timelike for a given detector (such as a simple atom), then there is a Lorentz frame in which field energy is transferred to the detector without a net transfer

[1] We will refer always to absorption of internal energy, although emission is obtained by interchanging the signs on the charges.

[2] Inasmuch as an elementary detector cannot simulate a blackbody, the amount of energy absorbed bears no necessary relation to the total energy in the field. The detector is only a yes-no indicator of the possibility of absorption. However, it will be shown that one can make some use of the magnitude of Δp_4.

of momentum. That is, the internal variables of the detector must have undergone a real transition corresponding to a change of total internal energy. On the other hand, if the impulse received is spacelike, then there is a Lorentz frame in which a net momentum is transferred to the detector without effecting its total energy. Although this may be accompanied by internal changes in the detector corresponding to a migration to degenerate states under the influence of the external field (as in the case of polarization), this is just the kind of process we mean to exclude by the criterion. The observable effects of such a process are always distinguishable from those transitions which absorb a net energy in all Lorentz frames.

Applying Stokes Theorem to (2.1) and using Maxwell's equations gives:

$$\begin{aligned} \Delta p_\mu &= \tfrac{1}{2} q \int_\Sigma (F_{\mu\beta,\alpha} - F_{\mu\alpha,\beta})\, d\sigma^{\alpha\beta} \\ &= \tfrac{1}{2} q \int_\Sigma F_{\alpha\beta,\mu}\, d\sigma^{\alpha\beta} \end{aligned} \tag{2.2}$$

where Σ is any surface of the loop.

Equation (2.2) shows that if the field $F_{\alpha\beta}$ is time independent, then whatever the geometry of the loop, Δp_4 must be zero. That is, no elementary detector can be constructed which will absorb internal energy from time independent electromagnetic fields in an inertial system.

The invariance of the absorption criterion permits it to be extended to noninertial frames of reference. However, Eq. (2.1) is not correct in more general frames, for the integrand at each point should be parallel displaced to a common point if Δp_μ is to be a four-vector. This is meaningful to do only when a uniquely definable displacement bitensor exists between any two points, as when the curvature of the space is zero, or when the loop is small enough to justify neglecting second order gravitational effects over its area.[3] If w is the common point, then the displacement bitensor $\Lambda_\mu{}^\kappa(w)$ will carry any four-vector A_κ at the field point into a four-vector $A_\mu(w) = \Lambda_\mu{}^\kappa(w)A_\kappa$ at point w. In a finite region of zero curvature, we calculate $\Lambda_\mu{}^\kappa(w)$ by first transforming A_κ to the local cartesian inertial frame where it is parallel displaced (without distortion) to point w, and there transforming it back to the original frame. That is, let $\Lambda_\mu{}^\kappa(w) = \alpha_\mu{}^\sigma(w)\bar{\alpha}_\sigma{}^\kappa$, where $\alpha_\mu{}^\sigma(w)$ is the transformation from the inertia frame to the given (gravitational) reference frame at point w.

Equations (2.1) and (2.2) now become:

$$\Delta p_\mu = q \oint \Lambda_\mu{}^\kappa(w) F_{\kappa\nu}\, d\lambda^\nu \tag{2.1a}$$

[3] Dewitt and Brehme (7) give a general treatment of bitensors showing their transformation properties, and define a bitensor of geodesic parallel displacement which can be applied in cases on nonzero curvature.

$$\Delta p_\mu = \tfrac{1}{2} q \int_\Sigma M_{\mu\alpha\beta}(w)\, d\sigma^{\alpha\beta} \tag{2.2a}$$

where

$$M_{\mu\alpha\beta}(w) = \Lambda_\mu{}^\kappa(w) F_{\alpha\beta,\kappa} + \Lambda_\mu{}^\kappa(w)_{,\alpha} F_{\kappa\beta} - \Lambda_\mu{}^\kappa(w)_{,\beta} F_{\kappa\alpha}$$

We see that time independent electromagnetic fields in conjunction with a static gravitational field does not generally guarantee a spacelike result in Δp_μ. An example of a timelike impulse arising in static fields is given below.

III. UNIFORMLY ACCELERATED FRAME

To be interesting for the problem at hand, a uniformly accelerated frame must have a time independent metric. The transformation which meets this requirement and which preserves the simplest special geometry (i.e., cartesian, and coinciding with the instantaneous inertial frame) is the transformation given by Møller (5). Let $x'^\mu = (z'x'y't')$ be the coordinates of the inertial frame, and $x^\mu = (zxyt)$ be the coordinates of the accelerated frame, where $t = c(\text{time})$. We want a charge e to have fixed coordinates x^μ and to have a constant proper acceleration in the $+z$ direction. For convenience, the charge is assigned coordinate values ($z = 1/a$, $x = y = 0$), where $a = (\text{const. proper accel.})/c^2$. If the two frames coincide with zero relative velocity at $t = 0$, then the Møller transformation equations are written:

$$z' = z \cosh \tau, \qquad x' = x, \qquad y' = y, \qquad t' = z \sinh \tau \tag{3.1}$$

where $\tau = at$. The metric in the accelerated frame is:

$$g_{11} = g_{22} = g_{33} = 1, \qquad g_{44} = -a^2z^2, \qquad g_{\mu\nu} = 0 \qquad (\mu \neq \nu) \tag{3.2}$$

The displacement bitensor from any point $(zxy\tau)$ on the accelerated frame to the point w is then given by:

$$\begin{aligned}
\Lambda_1{}^1(w) &= \cosh(\tau_w - \tau) & \Lambda_4{}^4(w) &= (z_w/z)\cosh(\tau_w - \tau) \\
\Lambda_4{}^1(w) &= az_w \sinh(\tau_w - \tau) & \Lambda_2{}^2(w) &= \Lambda_3{}^3(w) = 1 \\
\Lambda_1{}^4(w) &= (1/az_w)\sinh(\tau_w - \tau) & &\text{others equal to zero}
\end{aligned} \tag{3.3}$$

The electromagnetic fields produced by the charge e can be calculated in the $+z$ part of the accelerated frame from retarded potentials alone.[4] These were found by Born (8, 3, 4) to be:

$$\begin{aligned}
&E_z' = F'_{14} = \tfrac{1}{2}k[(z'^2 - \rho'^2 - t'^2 - 1/a^2]U' \qquad E_x' = F'_{24} = kz'x'U' \\
&E_y' = F'_{34} = kz'y'U' \qquad H_z' = F_{23} = 0 \qquad H_x' = F'_{31} = -ky't'U' \\
&H_y' = F'_{12} = kx't'U'
\end{aligned} \tag{3.4}$$

[4] The $+z$ and $-z$ parts of the accelerated frame are not continuously connected, but only one part is needed for comparison with an equivalence gravitational field.

where $F'_{\mu\nu}$ is the electromagnetic field tensor in the inertial frame, $k = 8ea^4$, $\rho'^2 = x'^2 + y'^2$, and

$$U' = \{a^4[1/a^2 - z'^2 - \rho'^2 + t'^2]^2 + 4a^2\rho'^2\}^{-3/2}$$

Using (3.1) to transform (3.4), the Born solutions on the accelerated frame are (6):

$$\begin{array}{lll} F_{14} = \frac{1}{2}akz[z^2 - \rho^2 - (1/a^2)]U & F_{24} = akz^2xU & F_{34} = akz^2yU \\ F_{24} = 0 & F_{31} = 0 & F_{12} = 0 \end{array} \qquad (3.5)$$

where $F_{\mu\nu}$ is the electromagnetic field tensor in the accelerated frame, $\rho^2 = x^2 + y^2$, and

$$U = \{a^4[(1/a^2) - z^2 - \rho^2]^2 + 4a^2\rho^2\}^{-3/2}$$

A static charge on this frame of static metric (3.2) therefore produces a time independent electromagnetic field with vanishing axial components.

We will now construct an elementary detector loop such that the first three components of Δp_μ evaluated in the accelerated frame are zero. Between end points A and B of the loop, a path is chosen which overlaps at a point C, thereby creating two subloops 1 and 2. The space projection of subloop 2 is made the same as that of subloop 1, but the projections of the two subloops into any space-time plane are made mirror images. With this choice of path, for each differential area $d\sigma^{\alpha\beta}(1)$ in subloop 1 there is a corresponding area $d\sigma^{\alpha\beta}(2)$ in subloop 2 such that:

$$d\sigma^{ij}(2) = d\sigma^{ij}(1); \qquad d\sigma^{i4}(2) = -d\sigma^{i4}(1) \qquad i, j = 1, 2, 3 \quad (3.6)$$

For convenience, let the space part of point w be $z_w = 1/a$, and let the time part be simultaneous with event C in the accelerated frame. The integrand $M_{\mu\alpha\beta}(w, 2)$ from (2.2a) evaluated at the area $d\sigma^{\alpha\beta}(2)$ is then simply related to the corresponding integrand in subloop 1.

$$\begin{array}{ll} M_{ijk}(w, 2) = -M_{ijk}(w, 1) & M_{ij4}(w, 2) = M_{ij4}(w, 1) \\ M_{4ij}(w, 2) = M_{4ij}(w, 2) & M_{4i4}(w, 2) = -M_{4i4}(w, 1) \\ & i, j, k = 1, 2, 3 \end{array} \qquad (3.7)$$

For this choice of path and point w, the impulse Δp_μ is found to be:

$$\Delta p_i = 0; \qquad \Delta p_4 = q\int_{\Sigma_1} M_{4ij}\, d\sigma^{ij} + 2q\int_{\Sigma_1} M_{4i4}\, d\sigma^{i4} \qquad (3.8)$$

Another choice of w would rotate the vector, but would not change its time-likeness which indicates that the system absorbs internal energy from the electromagnetic field. In the special case in which the loop is located in the x–y

plane at $z \approx 1/a$, and is in a weak (gravitational) field, $a \ll$ (dimensions of loop)$^{-1}$:

$$\Delta\mathcal{E} = -\Delta p_4 = 2qa \int_{\Sigma_1} F_{24}\, d\sigma^{21} \tag{3.9}$$

The loop path chosen above brings the charges back to their starting points on the accelerated frame, and so the process can be repeated indefinitely. It is therefore possible to construct a detector on the accelerated frame which continuously absorbs a timelike four-momentum from the electromagnetic field. Not only is a continuous change in total energy suggested by this result but, more important, a continuous time dependent detector behavior is suggested since local inertial observers will see real transitions induced in each cycle. The result raises the Bondi-Gold paradox in a slightly different form since the effect is local as well as possibly long range.[5]

To settle first the question of energy conservation, we recognize that the energy gained in a single cycle is absorbed unevenly over its path giving rise to a net displacement in the detector's center of mass, and that the accompanying change in gravitational energy is not included in our calculation of Δp_4. Consider the limiting case of a pair of charges moving in opposite directions around an area $dxdz$ which is placed in the x–z plane at $z \approx 1/a$. The field F_{i4} may be considered constant over this area since there will be no contribution from curl $\mathbf{E} = 0$, where $E_i = F_{i4}$. The energy gained by the two charges moving along the bottom of the area is $2qF_{24}\,|\,dx\,|$, and along the top it is $-2qF_{24}\,|\,dx\,|$ in the first approximation. The gravitational potential energy contained in this mass distribution is therefore:

$$-[2qF_{24}\,|\,dx\,|/c^2](ac^2)|\,dz\,| = -2qaF_{24}d\sigma^{21} \tag{3.10}$$

which cancels the term calculated in (3.9).

An exact treatment requires the identification of the conservative electromagnetic field quantity on the accelerated frame which is a natural extension of the field energy in an inertial frame. Following Møllers scheme (*9*), we identify:

$$F_{23} = B_z \qquad F_{31} = B_x \qquad F_{12} = B_y \qquad F_{14} = E_z \qquad F_{24} = E_x \qquad F_{34} = E_y$$

$$F^{23} = \frac{1}{az} H_z \qquad F^{31} = \frac{1}{az} H_x \qquad F^{12} = \frac{1}{az} H_y \qquad F^{14} = -\frac{1}{az} D_z$$

$$F^{24} = -\frac{1}{az} D_x \qquad F^{34} = -\frac{1}{az} D_y \qquad J^{\mu} = \frac{1}{az}(\mathbf{J}, c\rho)$$

[5] This result is ambiguous in so far as its being identified with radiation in the usual sense. The detector does not simulate a blackbody surface over which one can integrate at infinity; nor can we interpret the functional dependence in this metrical field without a blackbody comparison.

where $\mathbf{J}$ and ρ are current and charge densities relative to the accelerated frame. From the covariant form of Maxwell's equations, it can then be shown that the accelerated frame is isomorphic with a universe in which the dielectric constant and magnetic permeability of empty space are $\epsilon = \mu = 1/az$. The velocity of light and index of refraction are respectively azc and $1/az$, so that light rays will be deflected in the $-z$ direction as expected. In addition, one obtains the Poynting relation:

$$\frac{c}{4\pi}\operatorname{div}(\mathbf{E}\times\mathbf{H}) + \frac{1}{8\pi}\frac{\partial}{\partial t}\left(\frac{E^2 + H^2}{az}\right) + \mathbf{J}\cdot\mathbf{E} = 0 \tag{3.11}$$

which identifies the quantity associated with energy conservation in the accelerated system. The term $\mathbf{J}\cdot\mathbf{E}$ in (3.11) integrated over the volume of a charge q is equal to $-dp_4^{(\mathrm{accel})}/dt$, where $dp_4^{(\mathrm{accel})} = qF_{4\nu}d\lambda^\nu$. The net impulse $\Delta p_4^{(\mathrm{accel})}$ on an elementary detector loop is then to be found by integrating over the loop path without parallel displacement as given by Eq. (2.2). Since the fields $F_{\mu\nu}$ are time independent, it follows from (2.2) that $\Delta p_4^{(\mathrm{accel})} = 0$ for any loop configuration. There is therefore no violation of energy conservation in any region of the accelerated frame if gravitational potential is included. However, $\Delta p_\mu^{(\mathrm{accel})}$ is not a four-vector, and so the invariant criterion cannot apply to it.

The real problem is not in any case the discovery of formally conserved quantities in a gravitational field. Rather, it is the second apparent difficulty raised by the invariant calculation (3.8); for it is suggested by this timelike result that a stationary detector in a static gravitational and electromagnetic field will show a time dependent behavior of a kind associated with internal transitions to higher energy states. We can furthermore identify this behavior with a continuous lowering of the detector's center of mass. But then it is also clear that we have not fully satisfied the claimed conditions, for a detector cannot be said to be stationary if its center of mass is continually falling.

To satisfy definition, additional supports or constraints must be added to the detector system in the presence of a Coulomb source, where these perform the function of raising the center of mass in each cycle. Since these forces are not applied to the charge q directly, but only to the mass absorbed in each cycle, they are not formally included in (2.2a). However, they are crucial to recognize, for they oppose the internal transitions induced by the electric field, thereby establishing the required equilibrium. To the extent that they fulfill this requirement, their effect plus the effect of the electromagnetic forces must on average transfer either spacelike impulses to the detector system, or null impulses with zero components.

IV. THE ADDITIONAL CONSTRAINTS

One can analyze these forces in more general fields, where again we limit consideration to a region of zero space-time curvature. At each point λ on a detector loop, a momentum $Dp_\mu = qF_{\mu\nu}d\lambda^\nu$ is received by the detector system from the electromagnetic field. When this momentum is evaluated at another space-time point w it is equal to $Dp_\mu(w) = \Lambda_\mu{}^\kappa(w)Dp_\kappa$, where $\Lambda_\mu{}^\kappa(w)$ is the displacement bitensor. As before, the location of w will not affect the formal conclusion, but for clarity in this section we suppose it to be at the center of the mass distribution, and at a time which is quasistatically long after the completion of the given angle. Point w will then also be made the point to which all of the electromagnetic momentum contributions Dp_μ are physically transported by the additional support forces.

The support impulse required to lift Dp_μ from λ to w is given by the total change in Dp_μ, where both the initial value $Dp_\mu^{(i)}$ and the final value $Dp_\mu^{(f)}$ are evaluated at w. That is:

$$\begin{aligned} I(w) &= Dp_\mu^{(f)}(w) - \Lambda_\mu{}^\kappa(w)Dp_\kappa^{(i)}(w) \\ &= (Dp_\mu^{(f)}(w) - Dp_\mu^{(i)}(w)) + (\delta_\mu{}^\kappa - \Lambda_\mu{}^\kappa)Dp_\kappa^{(i)}(\lambda) \end{aligned} \tag{4.1}$$

For a differential displacement dx^δ from λ to w, the quantity $(\delta_\mu{}^\kappa - \Lambda_\mu{}^\kappa) = -\Gamma^\kappa_{\mu\delta}dx^\delta$, as assured by the definition of gamma (*10*).

When the lifting impulse (4.1) is added to the electromagnetic impulse $\Lambda_\mu{}^\kappa Dp_\kappa^{(i)}(\lambda)$. The total received from point λ on the detector loop becomes:

$$I(w) = (Dp^{(f)}(w) - Dp_\mu^{(i)}(\lambda)) + Dp_\mu^{(i)}(\lambda) \tag{4.2}$$

The value of (4.2) is conditioned by the further requirement that the lifting force along the path from λ to w makes no contribution to the momentum-energy other than that necessary to overcome gravitational force. This is equivalent to the usual quasistatic requirement. We know that the dynamical momentum-energy is given by a covariant four-momentum, since a covariant vector is obtained from the derivative of the action $\partial S/\partial x^\mu$. Hence, the quasistatic requirement states that the force at each point along the path is such that $d[Dp_\mu(s)] = 0$. When this is integrated over the path, we obtain the first term in (4.2) equal to zero. Therefore, the sum of the electromagnetic impulse and the additional support impulse coming from λ and evaluated at w is just $Dp_\mu^{(i)}(\lambda) = qF_{\mu\nu}d\lambda^\nu$. Integrating this over the loop gives:

$$\Delta p_\mu = \oint Dp_\mu(\lambda) = q\oint F_{\mu\nu}\,d\lambda^\nu = \tfrac{1}{2}q\int_\Sigma F_{\alpha\beta,\mu}\,d\sigma^{\alpha\beta} \tag{4.3}$$

which is identical with (2.2).

The four-vector Δp_μ is here understood to be evaluated at the point w where the loop momentum has been physically transported by the added constraints. The reason the rhs of (4.3) does not refer to w is that different lifting constraints are required for different choices of w, and in such a way that the numerical value of the resulting four-vector is the same for all w. That is, changing w here implies a physical change and not just a formal one.

In the special case of the static charge in a Møller accelerated frame, we have seen that the fields $F_{\alpha\beta}$ are time independent (3.5). Therefore, the fourth component of (4.3) will be $\Delta p_4 = 0$, assuring that a static elementary detector in such a field will not absorb internal energy in the sense of our criterion. The observational paradox is then removed by the addition of constraints which insure that the detector is stationary in the given reference frame.

V. FREE FALLING CHARGE

If an elementary detector were constructed in a Lorentz frame, and in the Coulomb field of a static electric charge, then it is clear from (4.3) that there could be no internal absorption. Suppose, however, the detector is accelerated uniformly in such a way as to make it stationary in a Møller accelerated frame. The difference between an accelerated and unaccelerated detector lies only in the constraints which are added to insure its being stationary in a preferred chosen frame. It is then a matter of interest to ask if such an accelerated detector can absorb internal energy from a static Coulomb field.

We must analyze this case from the accelerated frame using (4.3), where physically this case corresponds to asking if a stationary detector can absorb internal energy from a charge which is free falling in a gravitational field.

Let the charge e be fixed at the point $z' = 1/a$, $x' = y' = 0$ of the primed inertial frame. Then the electric field is given by:

$$E_z = F'_{14} = e(z' - 1/a)R'; \qquad E_x = F'_{24} = ex'R'$$

$$E_y = F'_{34} = ey'R'; \qquad R' = \{\rho'^2 + (z' - 1/a)^2\}^{-3/2} \qquad \rho'^2 = x'^2 + y'$$

From the accelerated frame reached by the transformation (3.1), the electromagnetic field tensors has values:

$$F_{12} = \sinh\tau\, F'_{42}\,; \qquad F_{13} = \sinh\tau\, F'_{43}$$

$$F_{14} = azF'_{14}\,; \qquad F_{24} = az\cosh\tau\, F'_{24}\,; \qquad F_{34} = az\cosh\tau\, F'_{34}$$

where in terms of the coordinates of the accelerated frame, the inertial frame fields are:

$$F'_{14} = ez(\cosh\tau - 1/az)R; \qquad F'_{24} = exR$$

$$F'_{34} = eyR; \qquad R = \{\rho^2 + (z\cosh\tau - 1/a)^2\}^{-3/2}$$

We now construct a special elementary detector loop consisting of two subloops similar to the one of Section III, where the corresponding areas satisfy (3.6). In this case, however, the loop is further specialized by requiring that event C, the event connecting the two subloops, occurs at time $\tau = 0$. Then the field derivatives at corresponding areas are related by:

$$F_{i4,j}(2) = F_{ij,j}(1); \qquad F_{ij,k}(2) = -F_{ij,k}(1) \tag{5.1}$$
$$F_{i4,4}(2) = -F_{i4,4}(1) \qquad F_{ij,4}(2) = F_{ij,4}(1)$$

Combining (3.6) and (5.1) with (4.3) yields:

$$\Delta p_i = 0; \qquad \Delta p_4 = q \oint_{\Sigma_1} F_{ij,4}\, d\sigma^{ij} + 2q \oint_{\Sigma_1} F_{i4,4}\, d\sigma^{i4} \tag{5.2}$$

It is therefore apparent that one can construct an elementary detector on a Møller accelerated frame which can absorb internal energy from a free falling charge. When described from the inertial frame of the charge, the same detector is of course seen as an accelerating detector in a static Coulomb field.

VI. SUMMARY

We have found that a stationary detector in a static electromagnetic field will not absorb internal energy, and that this result is independent of the existence of first order gravitational field. In particular, a static charge in a Møller accelerated frame will not induce internal transitions in a detector which is stationary in that frame. On the other hand, if a detector is accelerated through a static electromagnetic field, it can in general absorb internal energy. We have demonstrated this in Section V for the case of a detector which uniformly accelerates relative to an inertial frame containing a static electric charge. That is, a detector which is stationary in an accelerated frame will in general absorb internal energy from a free falling electric charge.

It is to be emphasized that the elementary detector loop which was used to establish an absorption criterion cannot simulate a blackbody, and so we cannot use it to confirm the results of others (*3*, *4*) that a uniformly accelerated charge or a free falling charge radiates a net energy, or to furnish magnitudes of radiation reaction. The present method is good only for investigating the consistency of such conclusion with possible invariant detector observations, and for investigating the effects of the constraining forces which physically attach detector to noninertial frames of reference.

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